Establishment of Quasi-Steady Conditions in a Blow-Down Nonequilibrium MHD Generator

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In the present paper, numerical calculations of a nonsteady nonequilibrium flow in a divergent linear MHD generator section are presented. This work is connected to an experimental program conducted by B. Zauderer of General Electric. The objective of this work was to find time necessary for the establishment of a steady-state condition at different generator loads and reservoir parameters. The working gas was argon, seeded with 1% of cesium. Results indicate that, as a general rule, the time needed to establish quasi-steady conditions is of the order of 2–3 msec, and if a secondary shock wave appears early, it remains also later at quasi-steady conditions.

Introduction

A LARGE steady-state MHD generator is quite expensive to build and to operate. It is much less expensive to perform research on a quasi-steady generator of the same scale. This was the basic idea behind the experimental program of B. Zauderer,[1] supported by O.N.R.

As an outgrowth of Zauderer's previous experiment with shock tube-driven MHD channel,[2] the quasi-steady blow-down facility consists of two reservoirs with a divergent MHD channel placed in between. The linear MHD generator has segmented electrode-wires separated from the walls to reduce boundary-layer effects. In this experiment, as in an earlier one, the flow is initiated by the rupture of a diaphragm. However, the main difference is the time scale and the fact that the working gas, in the latest facility, is a driver gas behind the contact surface. Another difference between the two experiments is that, in the nonsteady experiment, the temperature of the shock-heated gas was quite high (of the order of 11,000°K) to obtain high conductivity of the working gas. In the quasi-steady facility, the temperature of the expanding gas (argon) is relatively low, and the conductivity is provided entirely by the low ionization potential additive (cesium). Thanks to nonequilibrium flow with the electron temperature elevated over the gas temperature, the conductivity is sufficient to obtain interaction of the flow with the electromagnetic field. If the generator load is not too large and stagnation temperature in the reservoir is large enough, the interaction process can be very strong, resulting in the formation of a standing shock wave at the generator entrance.

Two questions naturally arise: 1) How long a time is needed to obtain a steady-state flow in such a generator? 2) If a secondary standing shock appears at the generator entrance at an early time, will it stay or disappear after a long time? This paper is devoted to answering these questions and giving a better insight into the nonsteady interaction processes leading to the establishment of quasi-steady flow with nonequilibrium ionization. This paper represents an extension of the author's previous works dealing with shock tube-driven MHD device[3] (corresponding to Zauderer's first experiment). The flow in the generator has been assumed to be nonsteady, one-dimensional in a channel with slowly variable cross-sectional area. The magnetic Reynolds number has been assumed to be small and, as a consequence, the induction effect has been neglected. The present calculations have been performed in the regime of a small Hall parameter so that the Hall current can be neglected. A larger Hall current would result in the appearance of a transverse Lorentz force pushing the plasma against the wall, making the flow non-one-dimensional. The electrode loss increases the effective generator load and can be incorporated in factor K [Eq. (7)].

Equations have been integrated using a modified two-step Lax-Wendroff numerical technique. This method is based on an equation in conservation form, and enables one to integrate across shock waves which are identified a posteriori as location with steep gradients. The ionization process has been assumed to be in equilibrium with the electron temperature and calculated from the Saha equation. The electron temperature was calculated from the energy equation representing the balance between joule heating and electron energy loss due to collision with heavy particles.

System of Equations and the Method of Solution

The system of equations representing conservation of mass, momentum, and energy, can be written as

$$W_t = F_x + S \tag{1}$$

with

$$W = A\begin{pmatrix} \rho \\ \rho u \\ \rho\left(\dfrac{u^2}{2} + e + I_i\right) \end{pmatrix}; \quad F = -A\begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u\left(\dfrac{u^2}{2} + h + I_i\right) \end{pmatrix}$$

$$S = A\begin{pmatrix} 0 \\ \dfrac{p}{A}\dfrac{dA}{dx} - j_y B_z \\ -Q + j_y E_y \end{pmatrix}$$

where t = time, x = space coordinate [subscripts in Eq. (1) denote derivatives], ρ denotes average gas density, u = velocity, p = pressure, e = internal energy, h = enthalpy, A = cross-sectional area, Q = rate of radiation loss [$Q = (4.0 \times 10^{-36} n_e / T_e^{1/2}) MKS$], j_y = electric current density, B_z = magnetic field, E_y = electric field in the direction perpendicular to the flow, $I_i = \alpha_s x_s (I_{is}/m) + \alpha_A (1 - x_s)(I_{iA}/m)$ denotes ionization energy per unit mass with α_s, α_A denoting degree of ionization of seed and argon, respectively, x_s = percentage of seed, and $I_{is} I_{ia}$ = ionization potential of seed and Argon. $m = (1 - x_s) m_A + x_s m_s =$

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Index categories: Plasma Dynamics and MHD; Electric Power Generation Research.

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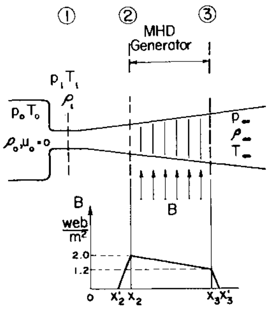

Fig. 1 Schematic of MHD generator system and magnetic field distribution.

average atomic mass of mixture (approximately equal to atomic mass of argon).

The electron temperature T_e can be calculated from the equation representing the balance between joule heating and the electron energy loss by collision with heavy particles

$$j_y^2/\sigma = \tfrac{3}{2}\delta_{ei}\, k(T_e - T)n_e v_{eq} \qquad (2)$$

where δ_{ei} denotes the energy transfer coefficient, ($\delta_{ei} = 2m_e/m$) is the Boltzmann constant, n_e denotes electron number density, v_{eq} is the collision frequency between electrons and heavy particles (mostly argon) given by

$$v_{eq} = \left(\frac{8kT}{\pi m_e}\right)^{1/2} \sum_q n_q Q_{eq} \qquad (3)$$

with $q = s,\ i,\ A$ denoting seed, ions, and argon, respectively. Electric conductivity σ can be written as

$$\sigma = e^2 n_e / m_e v_{eq} \qquad (4)$$

with e denoting electron charge; m_e is the electron mass.

The electron number density could be calculated for seed and argon from the Saha equation

$$\frac{n_{iq} n_e}{n_q} = \left(\frac{2\pi m_e k T_e}{h^2}\right)^{3/2} \frac{g_{iq} g_e}{g_q} \exp\left(-\frac{I_{iq}}{T_e}\right) \qquad (5)$$

where $q = s,\ A,\ h$ denotes the Planck constant, $g_{iq},\ g_e,\ g_q$ are statistical weights for ions, electrons, and neutrals, and I_{iq} denotes ionization potential.

This equation enables one to calculate the electron number density due to ionization of seeding material (cesium) as well as that of the working gas (argon). Numerical calculations indicate that for the average gas temperature $T < 6000°K$ argon ionization is negligibly small. For $T \approx 6000°K$, cesium is fully ionized and with increasing temperature, starting from 7000°, ionization of argon becomes important.

The electric current density is given by Ohm's law

$$j_y = \sigma(uB_z - E_y) \qquad (6)$$

or introducing load parameter K related to the outside circuit resistance R by (u_1 being the initial velocity)

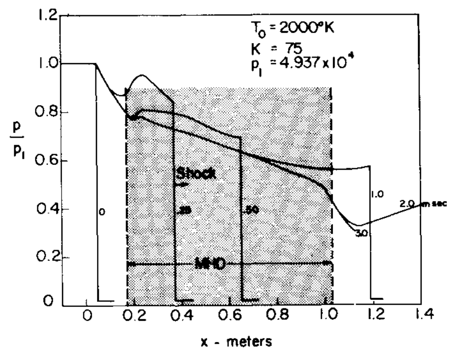

Fig. 3 Pressure distribution as a function of position and time for reservoir temperature $T_o = 2000°K$ and $K = 0.75$.

$$K = \frac{u\sigma RA}{l[1 + (\sigma RA/l)]u_1} \qquad (7)$$

where A is cross-sectional area, and l is a distance between electrodes, one obtains

$$E = Ku_1 B_z$$

and

$$j_y = \sigma u_1 B_z[(u/u_1) - K] \qquad (8)$$

The model of the channel has been taken as shown in Fig. 1. As a result of rapid change in the cross-sectional area from the reservoir to the channel, time derivatives could be neglected and flow is treated as quasi-steady. Parameters in the constant area channel in the front of the MHD generator or initial values behind the contact surface are given by

$$\begin{aligned} T_1 &= T_o\{1 + [(\gamma-1)/2]M_1^{\,2}\}^{-1} \\ p_1 &= p_o\{1 + [(\gamma-1)/2]M_1^{\,2}\}^{\gamma/\gamma-1} \end{aligned} \qquad (9)$$

where $M_1 = u_1/a_1$, $a_1 = (\gamma p_1/\rho_1)^{1/2}$. The subscript "$o$" denotes stagnation conditions in the reservoir.

The numerical method to solve the system of Eqs. (1) with additional Eqs. (2–9) is a two-step Lax-Wendroff finite-difference method similar to that used in Ref. 4. However, because of different electron and gas temperatures at each step, additional iteration procedure has to be used.

Results and Discussion

Calculations were performed for a divergent generator (Fig. 1) with the following cross-sectional areas: $A_1 = 0.014406$ m² (straight channel cross section), $A_2 = 0.01617$ m² (generator entrance), and $A_3 = 0.04826$ m² (generator exit). Magnetic field distribution is given by Fig. 1 with $x_2' = 0.17$ m, $x_2 = 0.226$ m, $x_3 = 0.976$ m, and $x_3' = 1.20$ m. At the entrance to the channel section of a constant area, one assumes constant uniform parameters. Such an assumption is valid under the condition that

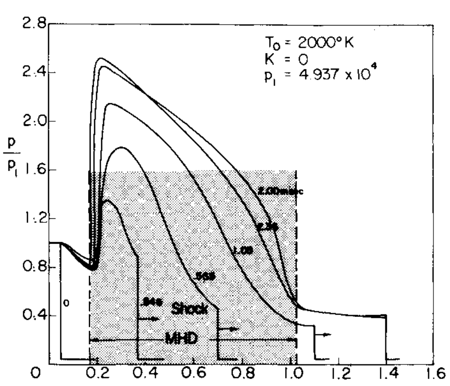

Fig. 2 Pressure distribution as a function of position and time (in msec) for reservoir temperature $T_o = 2000°K$ and $K = 0$ (short circuit).

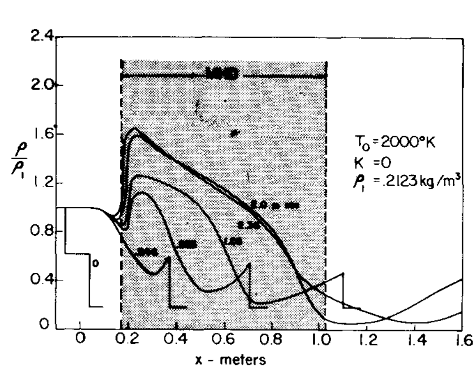

Fig. 4 Density distribution as a function of position and time for reservoir temperature $T_o = 2000°K$ and $K = 0$.

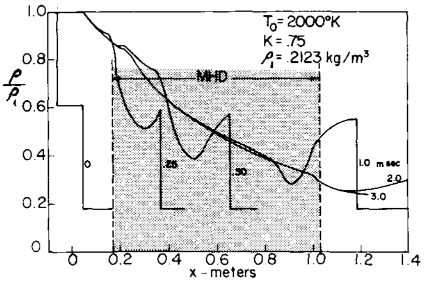

Fig. 5 Density distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0.75$.

flow is supersonic and conditions in the reservoir remain constant. Calculations show that flow parameters remain constant over the entire length of the channel with constant area, which indicates that there is no reflected shock from the generator section propagating upstream. This would be the case for a very strong interaction resulting in subsonic flow ahead of the generator. Interaction of the flow with the MHD generator section depends on load parameter K and reservoir conditions. Calculations were performed for $K = 0$ (short circuit), $K = 0.4$, and $K = 0.75$, and different stagnation temperatures in the reservoir T_o. Obviously, the strongest interaction corresponds to $K = 0$ and to the highest value of T_o.

It was assumed that the contact surface is relatively close to the shock wave with the driven and driver gas being the same. Results of the calculations are presented in Figs. 2–12. Salient characteristics of the interaction are as follows:

The time of establishment of a steady-state condition is of the order of $2 \div 3$ msec with the shorter time for larger generator loads and lower reservoir temperature T_o. In other words, for a weaker interaction, the steady-state conditions are being reached earlier. The peak of temperature behind the primary shock (Fig. 8) results from shock-heated driven gas. This peak disappears as shocks move out of the interaction zone.

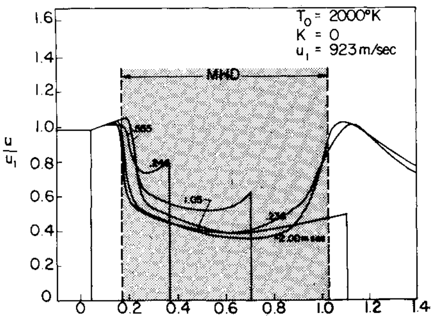

Fig. 6 Velocity distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0$.

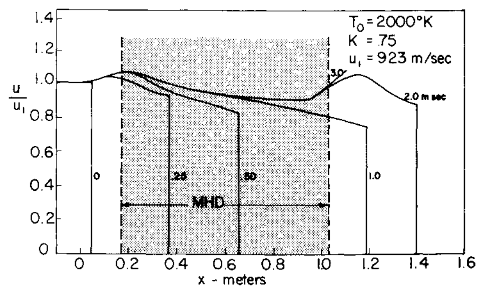

Fig. 7 Velocity distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0.75$.

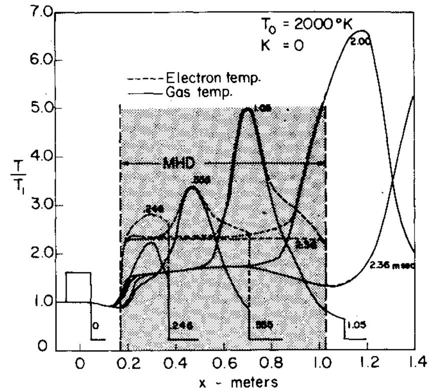

Fig. 8 Gas and electron temperature distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0$.

One can notice that, for a strong interaction, a secondary stationary shock wave is formed (Figs. 2, 4, and 6) near the generator entrance. This wave becomes stronger as time goes on, reaching maximum strength at the quasi-steady condition, and never disappears. However, for the weak interaction (Figs. 3, 5, and 7), no secondary shock wave was formed.

For the weak interaction (Fig. 9), the electron temperature at quasi-steady state is always elevated over the gas temperature. At the nonsteady conditions at peak temperatures in the driven gas behind the moving shock, locally, the gas temperature becomes equal to the electron temperature (Fig. 8). At the

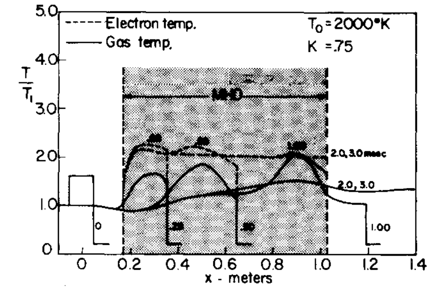

Fig. 9 Gas and electron temperature distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0.75$.

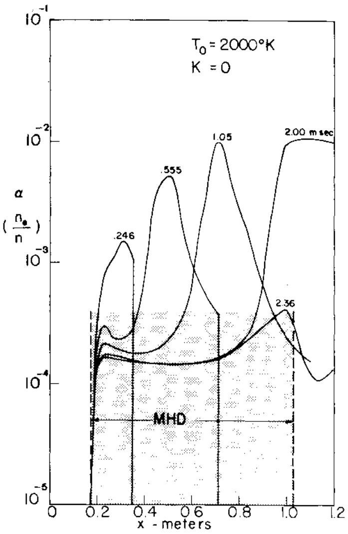

Fig. 10 Distribution of degree of ionization as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0$.

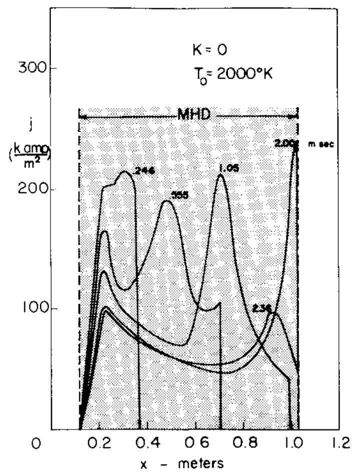

Fig. 11 Current density distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0$.

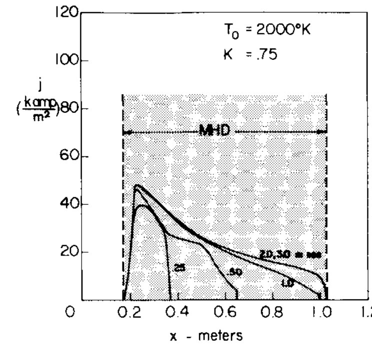

Fig. 12 Current density distribution as a function of position and time for reservoir temperature $T_o = 2000$°K and $K = 0.75$.

strongest interaction (Fig. 10), cesium locally becomes fully ionized with a small percentage of argon ionization. For weaker interaction, ionization is due to cesium only.

Current density distribution shows double peaks—one behind the shock wave and the second at the generator entrance (Figs. 11, 12). At quasi-steady conditions, there is a large current density peak at the generator entrance with a disappearing second current density peak near the exit. Such current density has the same character as that obtained experimentally by Zauderer.[1] The maximum current density near the generator entrance is exaggerated by the divergent channel and the decreasing, downstream magnetic field. Similar distribution was obtained for a constant cross-sectional area generator and a uniform magnetic field.

The estimated temperature drop, near the wall, due to the heat conduction in the time of 3 msec, is less than 100°K and should not result in significant non-one-dimensional effects. Preliminary experimental results[5] in a short channel show the formation of oblique rather than normal shocks, indicating a non-one-dimensional flow. It is believed that for a longer electrode section, the flow will be one-dimensional with a normal shock. It must be pointed out that the present calculations were obtained preceding any quantitative experimental results in a nonequilibrium quasi-steady MHD generator, and it is too early to compare them with the experimental results.

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